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Advanced Photon Source

# Flat Beam Generation

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and

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**26th Advanced ICFA Beam Dynamics  
Workshop on Nanometre-Size Colliding Beams**  
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# Flat Beam Generation

Beam adapter:

Ya. Derbenev, A. Burov, S. Nagaitsev

*Theory; Burov, Nagaitsev & Derbenev*

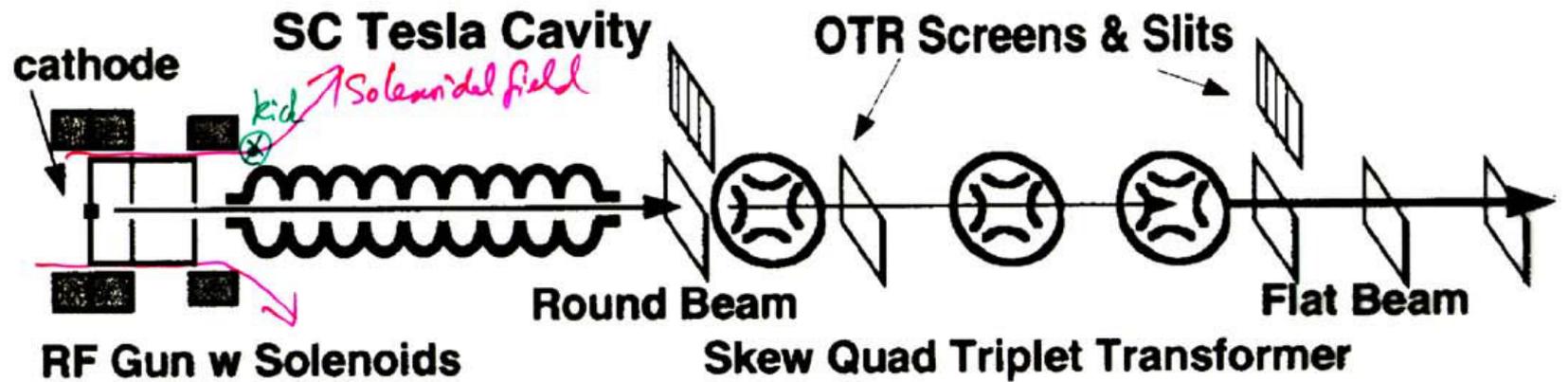
Application to flat beam:

R. Brinkmann, Ya. Derbenev, and K. Floetmann

Experiment:

FNPL at Fermilab

D. Edwards, H. Edwards, et al.

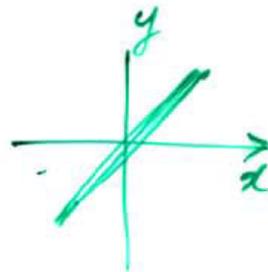


Schematic rendition of the layout at Fermilab for flat beam experiment.



$$\begin{cases} x = A \cos \kappa z \\ y = A \sin \kappa z \end{cases}$$

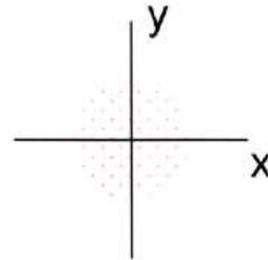
$$\rightarrow \begin{cases} A \cos \kappa z \\ A \sin(\kappa z + \pi/2) \\ \cos \kappa z \end{cases}$$



# Phase Space Coordinates

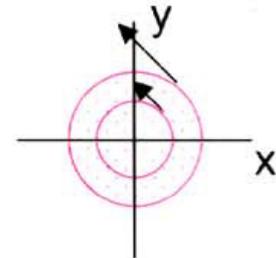
I. After cathode

$$X_I = (x, x', y, y')$$



II. After solenoid (short)

$$X_{II} = (x, x' + \kappa y, y, y' - \kappa x'), \quad \kappa = \left( \frac{eB}{2P_s} \right)$$



*( $\frac{1}{2}$  of Larmor frequency)*

### III. After quadrupole channel

$$X_{III} = R^{-1} \begin{bmatrix} M, & 0 \\ 0 & FM \end{bmatrix} R \cdot X_{II}$$

$$R = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}, M = \begin{bmatrix} \cos \mu, & \beta \sin \mu \\ -(\sin \mu)/\beta, & \cos \mu \end{bmatrix}, F = \begin{bmatrix} 0, & \beta \\ -\frac{1}{\beta}, & 0 \end{bmatrix}$$

↓  
skew

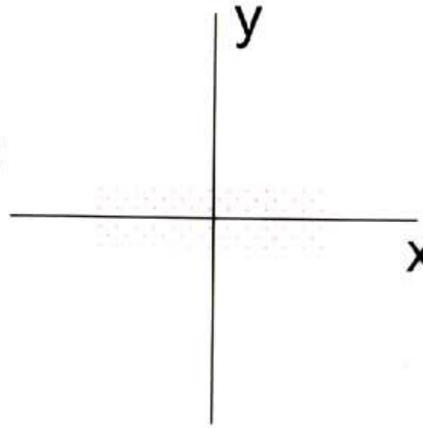
↓  
quad transport

↓  
 $+(\pi/2)$

Choose  $\beta$  (match)

$$X_{IV} = \begin{pmatrix} 2bx + 2ay + \beta(ax' - by') \\ -\frac{2a}{\beta}x + \frac{2b}{\beta}y + bx' + ay' \\ -\beta(bx' - ay') \\ ax' + by' \end{pmatrix} \text{ for } \beta = \kappa^{-1}$$

*→ 0 for  $x' = y' = 0$*



$$a = \frac{1}{2}(\cos\mu + \sin\mu), \quad b = \frac{1}{2}(\cos\mu - \sin\mu)$$

Flat beam for  $x' = y' = 0!$

# Beam Moment Matrix

$$\Sigma = \langle \mathbf{X}\mathbf{X}^T \rangle = \begin{pmatrix} \langle x^2 \rangle, \langle xx' \rangle, \langle xy \rangle, \langle xy' \rangle \\ \langle x'x \rangle, \langle x'^2 \rangle, \langle x'y \rangle, \langle x'y' \rangle \\ \langle yx \rangle, \langle yx' \rangle, \langle y^2 \rangle, \langle yy' \rangle \\ \langle y'x \rangle, \langle y'x' \rangle, \langle y'y \rangle, \langle y'^2 \rangle \end{pmatrix}$$

$$\Sigma_1 = \begin{pmatrix} \sigma^2 & 0 & 0 & 0 \\ 0 & \sigma'^2 & 0 & 0 \\ 0 & 0 & \sigma^2 & 0 \\ 0 & 0 & 0 & \sigma'^2 \end{pmatrix} \text{ (round beam, no correlation)}$$

$$\text{Det}(\Sigma_1) = (\sigma\sigma')^2$$

$$\Sigma_{||} = \begin{pmatrix} \sigma^2 & 0 & 0 & -\kappa\sigma^2 \\ 0 & \sigma'^2 + \kappa^2\sigma^2 & \kappa\sigma^2 & 0 \\ 0 & \kappa\sigma^2 & \sigma^2 & 0 \\ -\kappa\sigma^2 & 0 & 0 & \sigma'^2 + \kappa^2\sigma^2 \end{pmatrix}$$

$$\text{Det} (\Sigma_{||}) = (\sigma\sigma')^2$$

of  $x-x'$  only

$$E_x \approx \sigma \sqrt{\sigma'^2 + \kappa^2\sigma^2}$$

(this is not really emittance)

Choosing  $\beta = \alpha^{-1}$

$$\Sigma_{III} = \begin{bmatrix} \epsilon_x \mathbf{T} & \epsilon_{xy} \mathbf{S} \\ \epsilon_{xy} \mathbf{S} & \epsilon_y \mathbf{T} \end{bmatrix}$$

$$\epsilon_x = \frac{2\sigma^2}{\beta} + \frac{\beta\sigma'^2}{2}, \quad \epsilon_y = \frac{\beta}{2}\sigma'^2, \quad \mathbf{T} = \begin{bmatrix} \beta & 0 \\ 0 & 1/\beta \end{bmatrix}, \quad \epsilon_{xy} = \frac{\beta\sigma'^2}{2}$$

$$\mathbf{S} = \begin{bmatrix} -\beta \cos \mu & \sin 2\mu \\ \sin 2\mu & \frac{\cos \mu}{\beta} \end{bmatrix}$$

$$\text{Det } \mathbf{S} = -1$$

$$\text{Det } \Sigma_{III} = \epsilon_x \epsilon_y - (\epsilon_{xy})^2 = (\sigma \cdot \sigma')^2$$

Choose  $1 - \alpha^2 \beta^2 = \frac{\beta^2 \sigma'^2}{\sigma^2}$

$$\Sigma = \frac{1}{2} \begin{bmatrix} \beta E_x & 0 & 0 & 0 \\ 0 & E_x/\beta & 0 & 0 \\ 0 & 0 & \beta E_y & 0 \\ 0 & 0 & 0 & E_y/\beta \end{bmatrix} \quad !!$$

$$E_x = \frac{1}{2} \left[ \frac{(1 + \alpha\beta)^2}{\beta} \sigma^2 + \beta \sigma'^2 \right]$$

$$E_y = \frac{1}{2} \left[ \frac{(1 - \alpha\beta)^2}{\beta} \sigma^2 + \beta \sigma'^2 \right]$$

$$E_x E_y = (\sigma \sigma')^2$$

div.  $\frac{E_x}{E_y} = \frac{(1 + \beta\alpha)^2 \sigma^2 + \beta^2 \sigma'^2}{(1 - \beta\alpha)^2 \sigma^2 + \beta^2 \sigma'^2}$

Choose  $r = \left(\frac{\beta \sigma'}{\sigma}\right)^2 \ll 1$

$$\frac{E_x}{E_y} \Rightarrow \frac{4\sigma^2}{\sigma'^2}$$

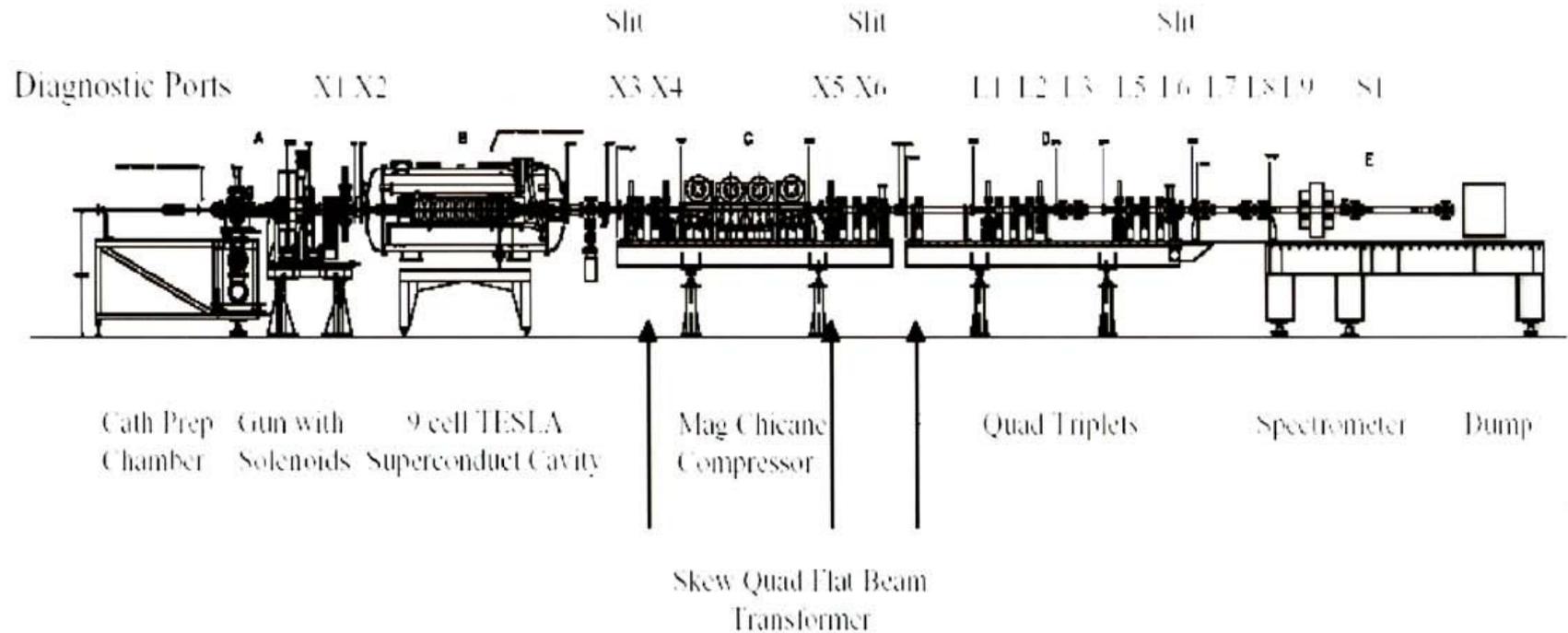
flat beam ratio  
(BDF)

	TESLA	NLC	CLIC
$Q(nc)$	3	1.6	0.6
$E_x$ (mm-mrad)	12	4.5	0.68
$E_y$	0.03	0.1	0.02
$\sqrt{E_x E_y}$	0.6	0.7	0.12

Current state of art

$$\sqrt{E_x E_y} \sim 1 \text{ mm-mrad} @ Q \approx 1 \text{ nC.}$$

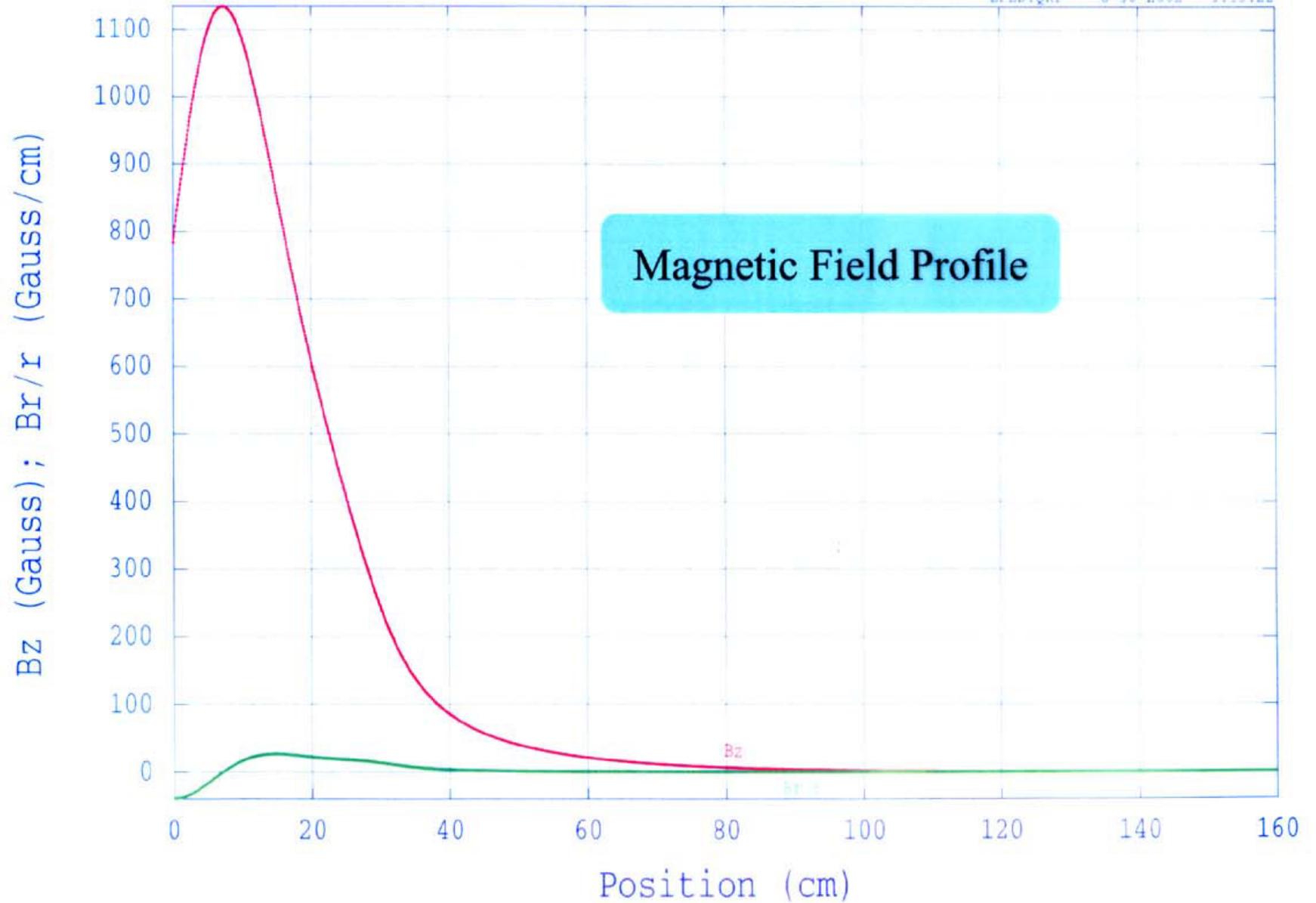
# FermiLab/NICADD PhotoInjector



Layout taken from PAC01 paper of D. Edwards etc.

Solenoid Magnet with Bucking Coil: main=170, bucker=0, 2nd=70

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## Cathode and beam prior to the skew-quad channel



Virtual Cathode (VC)



X3



X4 slit image



X5 slit image



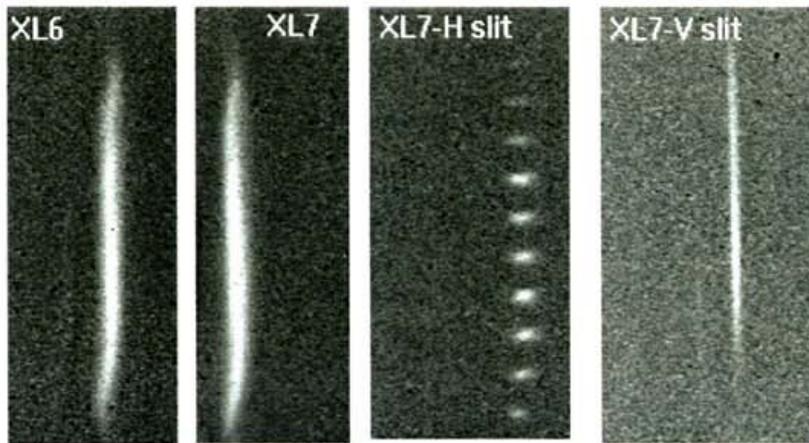
XS4

RMS VC	RMS beam size at X3	Normalized Emittance
0.73mm	1.66mm	6.44 mm mrad

VC is an image of the laser spot on the cathode.

Beam energy at the exit of the 1.6 RF gun is about 4 MeV. After the 9-cell SC cavity, beam energy is about 16 MeV. Notice that X3 is after the 9-cell SC cavity.

Flat electron beam profile at 9.6m from the cathode (XL6) and horizontal and vertical beamlets used for emittance measurements downstream at XL7 and XL8. The transverse emittance ratio is about 41 in the example shown here.



$$\frac{\epsilon_n^y}{\epsilon_n^x} = \frac{38.85 \text{ mm mrad}}{0.95 \text{ mm mrad}} = 41$$

