LOW LEVEL RF FEEDBACK LOOP DESIGN

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Abstract

Designing feedback loops for a circular particle accelerator low level RF can be based on a state variable representation. It includes the design of phase, radial feedback loops as well as synchronization loops between an injector and the main accelerator. A state space model linking the main variables typically used to describe a RF system is introduced before presenting the feedback design.

Keywords*:* Radiofrequency, feedback systems, state variable approach, pole assignment

1. INTRODUCTION

A state space description can be developed to model the evolution of the main variables used to describe the radiofrequency system of a circular particle accelerator. That description leads to the design of a feedback system based on a pole placement approach, providing greater stability and smaller errors.

2. DESCRIPTION OF THE LOOPS

2.1 Main variables and transfer functions

The main variables use to describe the system are [1]:

φ the instantaneous phase deviation of the bunch from the synchronous phase.

δR the variations of the beam radius

ω_{rf} the RF frequency

$\delta\omega_b$ the variations of the beam frequency

φ_b the phase of the beam with respect to the RF

These variables are linked through the following transfer functions:

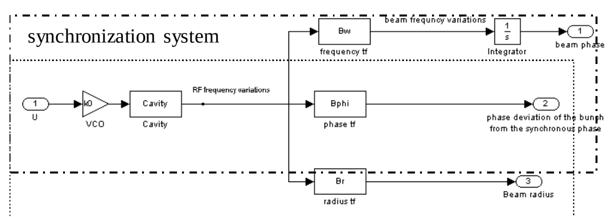
$$B_\varphi(s) = \frac{\varphi}{\delta\omega_{rf}} = \frac{s}{s^2+\omega_s^2}$$

$$B_R(s) = \frac{R}{\delta\omega_{rf}} = \frac{b}{s^2+\omega_s^2}$$

$$B_\omega(s) = \frac{\delta\omega_b}{\delta\omega_{rf}} = \frac{s}{s^2+\omega_s^2}$$

where b is a scaling factor and ω_s the synchronous frequency.

Figure shows the model of the system. K_0 denotes the gain of a voltage control oscillator and the cavity the transfer function of the accelerating cavity assumed to be unity.



Phase and radial system

Figure 1: System model

2.2 Phase and radial loop

<u>*2.2.1 Basic concept*</u>

To design the phase and radial loop, the phase transfer function B_φ and the radius transfer function B_R have to be taken into account.

It corresponds to the following system:

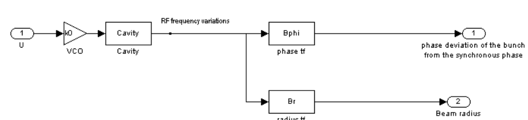


Figure 1: Phase and radial system

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This system can be described using two state variables, the first one being R/b and the second one being the phase φ as shown in :

$$\begin{cases} x_1 = \dfrac{R}{b} = \dfrac{k_0}{s^2 + \omega_s^2} U \\ x_2 = \varphi = s x_1 \end{cases} \qquad (1)$$

leading to the state space representation:

$$\frac{d}{dt}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \underbrace{\begin{pmatrix} 0 & 1 \\ -\omega_s^2 & 0 \end{pmatrix}}_{A_s}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \underbrace{\begin{pmatrix} 0 \\ k_0 \end{pmatrix}}_{B_s} U$$
$$y = \begin{pmatrix} \varphi \\ R \end{pmatrix} = \underbrace{\begin{pmatrix} 0 & 1 \\ b & 0 \end{pmatrix}}_{C_s}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \underbrace{\begin{pmatrix} 0 \\ 0 \end{pmatrix}}_{D_s} U \qquad (2)$$

Equation (2) defines the continuous state space representation [A_s, B_s, C_s, D_s].

From this continuous representation, a discrete state space representation (A_{sdiscr}, B_{sdiscr}, C_{sdiscr}, D_{sdiscr}) can be determined, using an exact zero order hold discretization.

One more state, the integral of the radius error “interror”, will be added to that representation in order to eliminate the static error:

$$\text{interror}_{n+1} = \text{interror}_n + \left(R_{ref} - R\right) = \mathbf{Z} \qquad (3)$$

The final state matrix is:

$$F = \begin{bmatrix} A_{sdiscr} & 0 \\ -C_{sdiscr} & 1 \end{bmatrix} \qquad (4)$$

and the final command matrix is:

$$H = \begin{bmatrix} B_{sdiscr} \\ 0 \end{bmatrix} \qquad (5)$$

where the observation matrix is:

$$\begin{bmatrix} C_{sdiscr} \\ 0 \end{bmatrix} \qquad (6)$$

2.2.2 Controller determination

Using the discrete state space representation established in paragraph 0, a set of feedback gains [2] $\lfloor K_\varphi \quad K_R \quad K_{\text{int}} \rfloor$ will be determined using pole placement so that the feedback signal is:

$$U = -\left(K_\varphi \varphi + K_R R - K_{\text{int}} Z\right) \qquad \textbf{(7).}$$

A pole placement approach has been chosen for the easiness to determine the feedback gains.
The closed loop poles are selected as poles of a Bessel filter because of its damping factor of 0.86 and its easiness to determine. The closed loop bandwidth is chosen to be at least $\omega_{cl} = 2 \cdot \pi \cdot 200$ rad/s to have a 20ms settling time. The closed loop bandwidth determines how fast the system responds.
The bandwidth of the Bessel filter is related to its settling time by [3]:

$$\omega_{cl} = 2 * \pi * \frac{0.704}{t_{settling}} \qquad (8)$$

A Bessel filter dynamic is chosen because of its non overshooting behavior.

The controller has been implemented in a cascaded structure, as shown in Figure 2.

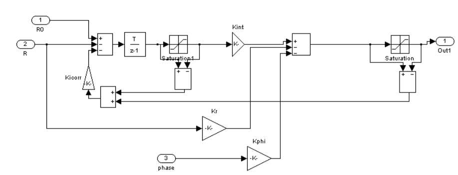

Figure 2: Feedback in a cascaded structure

2.2.3 Closed Loop results

The following figures show the phase and radial loop response.

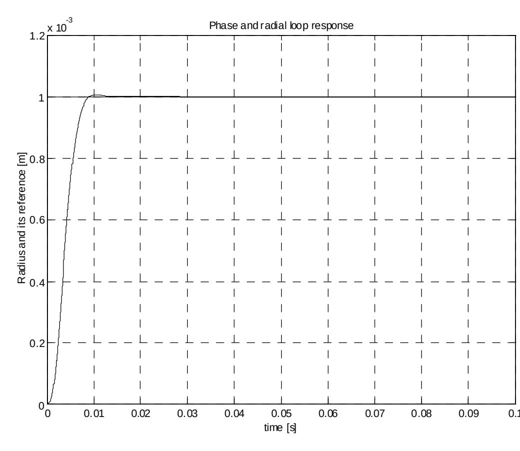

Figure 3: Simulation results for a 1 mm radial step, radius response

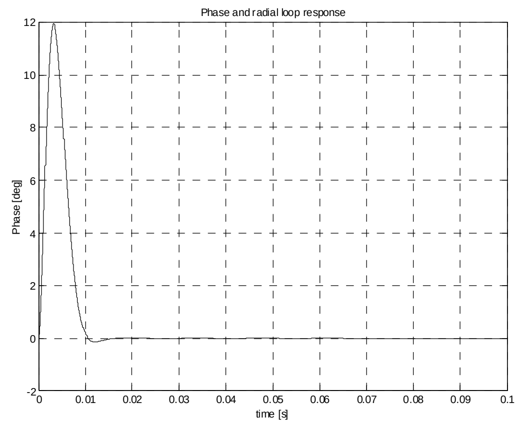

Figure 4: Simulation results for a 1 mm radial step, phase response

The radius and the phase reach their final value in 1ms. The following figure gives the system and its controller open loop Bode plot which confirms the 200 Hz controller bandwidth.

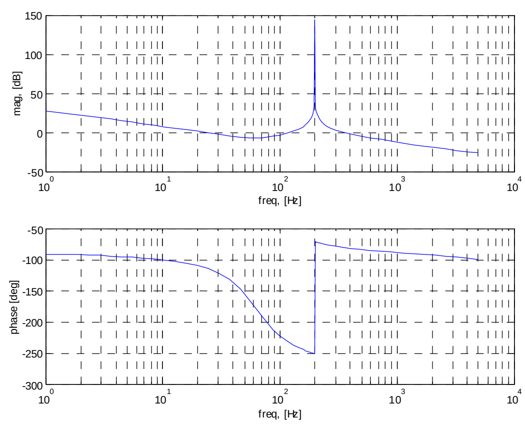

Figure 5: System with controller open loop Bode plot

From Figure 5 one can read a gain margin of 10 dB (it will depend on the beam damping) and a phase margin of 80 degrees.

2.3 *Digital Synchronization loop*

2.3.1 Basic concept

Synchronization means that the beam, and thus the rf, is rigidly phased (on average) with the reference:

$$\varphi_b - \varphi_{ref} = \varphi_{set} \qquad (9)$$

The value of φ_{ref} and φ_{rf} (and φ_b) is incremented at each clock cycle (the clock is supposed to be common). If the reference and the beam were at the same frequency, the output of the phase subtracting point would be constant. In the general case it will be a saw tooth with a repetition rate equal to the frequency difference. The output of the phase comparator can be forced to a constant value during acceleration by using an offset signal.

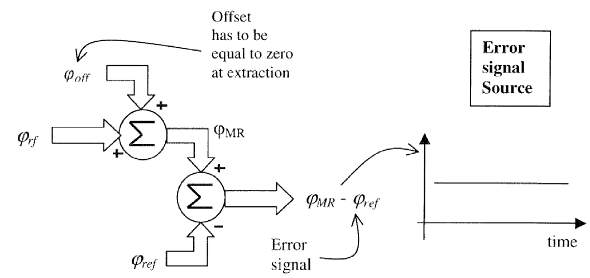

Figure 6: Introduction of an offset signal for a constant error signal

The error signal is now the difference of an extrapolated (or estimated) rf phase φ_{MR} (the moving reference phase) and the reference phase φ_{ref}. The principle is that the increment per clock period of φ_{off} plus the increment of φ_{rf} is equal to the increment of φ_{ref}. The resulting φ_{MR} - φ_{ref} is thus constant during the accelerating cycle. This trick enables the synchronization loop to be closed at any time.

The synchronization loop being closed, the performance of the mechanism will entirely rely on how you bring the moving reference phase to zero.

To design the synchronization loop, the phase transfer function B_φ and the frequency transfer function B_w have to be taken into account.
It corresponds to the following system:

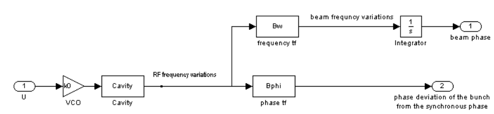

Figure 7: Synchronization system

This subsystem can be described using three state variables, the first one being φ_b and the second one being the beam frequency ω_b and the third one the phase φ.

$$\begin{cases} x_1 = \varphi_b = \dfrac{1}{s}\omega_b \\ x_2 = \omega_b = \dfrac{\omega_s^2}{s^2+\omega_s^2}\omega_{rf} \\ x_3 = \varphi = \dfrac{s}{s^2+\omega_s^2}\omega_{rf} \end{cases} \quad (10)$$

leading to the state space representation:

$$\frac{d}{dt}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \underbrace{\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & \omega_s^2 \\ 0 & -1 & 0 \end{pmatrix}}_{A_s}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \underbrace{\begin{pmatrix} 0 \\ 0 \\ k_0 \end{pmatrix}}_{B_s} U$$

$$y = \begin{pmatrix} \varphi_b \\ \omega_b \\ \varphi \end{pmatrix} = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{C_s}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \underbrace{\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}}_{D_s} U \quad (11)$$

Equation (2) defines the continuous state space representation [A_s, B_s, C_s, D_s].

From this continuous representation, a discrete state space representation (A_{sdiscr}, B_{sdiscr}, C_{sdiscr}, D_{sdiscr}) can be determined, using an exact zero order hold discretization.

One more state, the integral of the phase error is added.

2.3.2 Controller determination

Using the discrete state space representation established in paragraph 0, a set of feedback gains $\lfloor K_{\varphi_b} \quad K_{\omega_b} \quad K_{\varphi} \quad K_{\text{int}} \rfloor$ will be determined using pole placement so that the feedback signal is:

$$U = -\left(K_{\varphi_b}\varphi_b + K_{\omega_b}\omega_b + K_{\varphi}\varphi - K_{\text{int}}\int\left(\varphi_{ref} - \varphi_b\right)\right)$$

(12).

The poles are again chosen to be the poles of a 200 Hz third order Bessel filter.
The controller has been implemented in a cascaded structure, as shown in Figure 2.

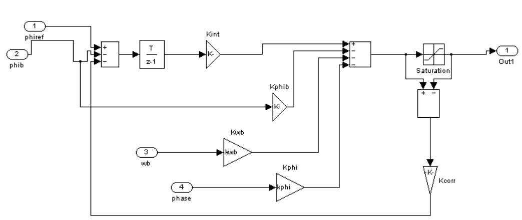

Figure 8: Feedback in a cascaded structure

2.3.3 Closed Loop results

The following figure shows the synchronization loop response.

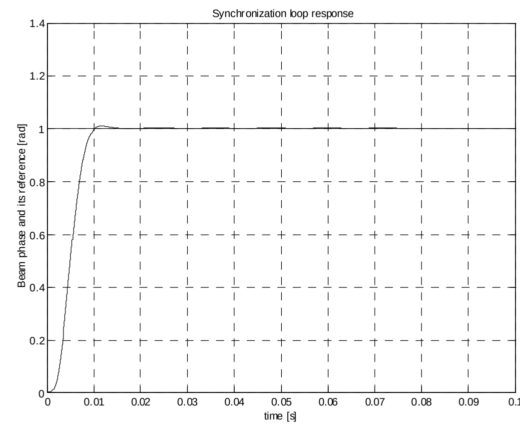

Figure 9: Synchronization loop response

Figure 10. gives the system and its controller open loop Bode plot which confirms the 200 Hz controller bandwidth.

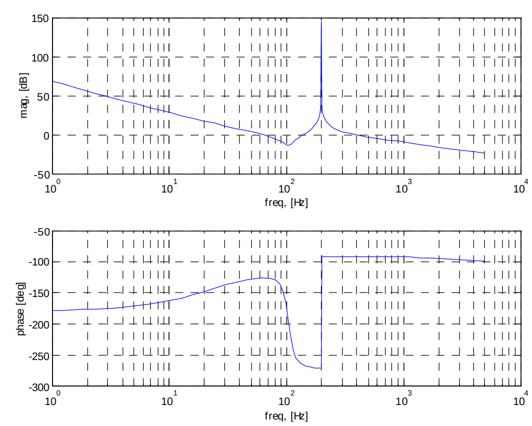

Figure 10. System with controller open loop Bode plot

From Figure 10, one can read a gain margin of 15 dB (it will depend on the beam damping) and a phase margin of 80 degrees.

3. CONCLUSION

The use of a state space representation leads to an easy design of two feedback loops, which provide good stability and performance. Moreover, the feedback gains could be calculated so as to take into account the accelerator parameter variations.

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