Coherent Synchrotron Radiation Effect in the NLC Damping Ring

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for

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Motivation

- **Stupakov-Heifets theory [PRST-AB, 5(2002)054402] indicates a potential instability due to the CSR in dipoles;**

- **Experimental observations of bursting IR radiation**
  - **John Byrd, *et al.* , ALS, LBNL**
  - **Larry Carr, Jim Murphy, *et al.* , NSLS, BNL**
  - **Bessy-II**

Review of S-H theory

1-D Vlasov Equation:

$$\frac{\partial r}{\partial s} - hd\frac{\partial r}{\partial z} - \frac{r_0}{g}\frac{\partial r}{\partial d}\int_{-\infty}^{\infty} dz' dd' w(z-z')\, r(d', z', s) = 0$$

Distribution Function

$$r = r_0(d) + r_1(d, z, s)$$

and

$$r_1 = \hat{r}_1 e^{-iws/c + ikz} = \hat{r}_1 e^{-i\,\mathrm{Re}(w)s/c + \mathrm{Im}(w)s/c + ikz}$$

Im(w)>0 means instability

Impedance Z(k)

- **Dipole**

$$Z(k) = iA \frac{k^{1/3}}{R^{2/3}}$$

with

$$A = 3^{-1/3}\Gamma\left(\frac{2}{3}\right)(\sqrt{3}i - 1)$$

Detailed study has been done by G. Stupakov and S. Heifets, [G. Stupakov & S. Heifets, PRST-AB, 5(2002)054402]

Consideration

- Energy modulation $Z(k) \sim k^{1/3}$

- The energy spread and momentum compaction smears this process and stabilized the instability. This 'damping' effect scales as k
- Two competing processes k vs. $k^{1/3}$ => fastest growth at smaller k.

Threshold is determined by the longest wavelength allowed by the pipe cut-off l ~ 1 mm

CSR effects in a wiggler

An estimate

- **Approximate the wiggler as 2N pieces of Dipoles**

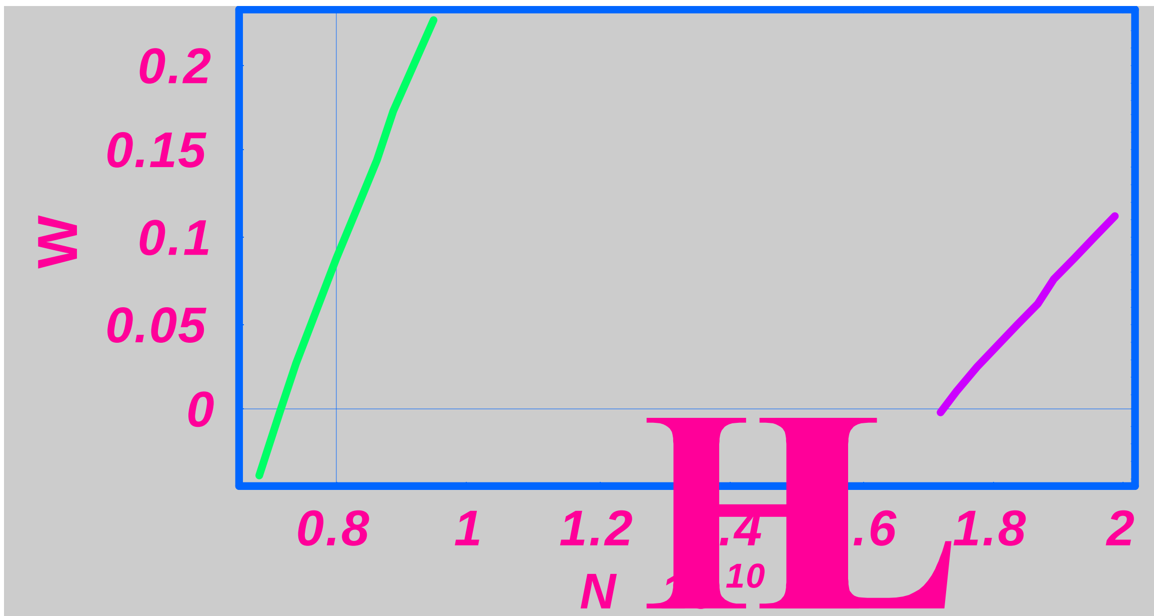
$$R(s)^{-1} = \frac{k_w K \left|\cos(k_w s)\right|}{g}$$

average

$$\overline{R}^{-1} = \frac{2}{p} \frac{k_w K}{g}$$

Results of Dipole and Wiggler CSR at pipe cut-off frequency

- Threshold drops from 1.75×10^{10} to 7.25×10^{9}



It is serious!!! Recall the design value is 7.5×10^{9};

Is bending magnet approach valid?

Characteristic Length

- **Incoherent Synchrotron Radiation Critical Wavelength: 0.6 Å;**
- **Cooperative/formation length for a radiation wavelength l_f**
  - **Dipole:** $L_f \sim \sqrt[3]{24R^2 l_f}$
  - **Wiggler:** $l_f \sim l_{\mathrm{FEL}} \sim 13mm \Rightarrow L_f$ **~ Wiggler period**

For λ shorter than FEL wavelength → critical wavelength, we could model the Wiggler as 2N pieces of Dipoles **but** for longer need to include interference

Wake Potential

Approach II

- **Focus on the averaged long-range wake potential. [E.L. Saldin, E.A. Schneidmiller & M.V. Yurkov, NIMA 417(1998)158]**

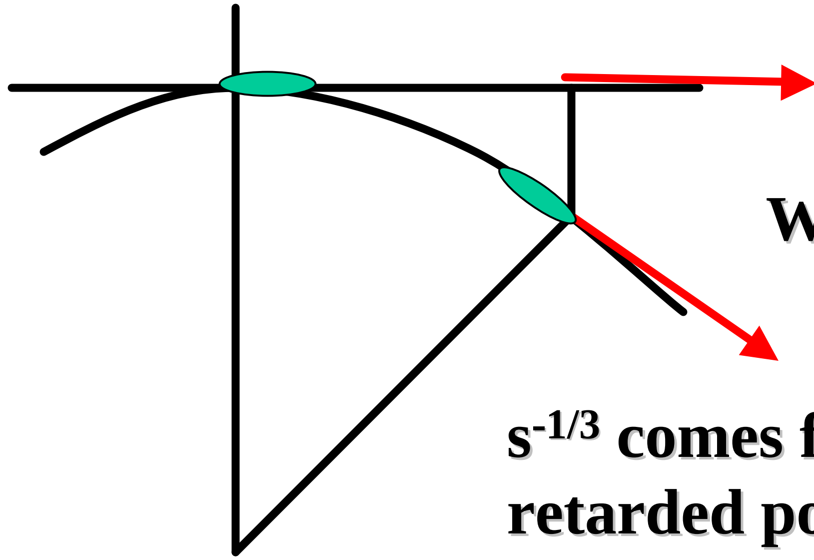
$$\overline{W}_W(z) = -k_W \frac{K^2}{2g^2 k_W z}\left[2\sin^2\left(\frac{2g^2 k_W z}{K^2}\right) + \frac{1}{K}\right]$$

Is it reasonable? It only contains the contribution from the fundamental and scales as 1/s at small s

Physics Model

- **Dipole:**

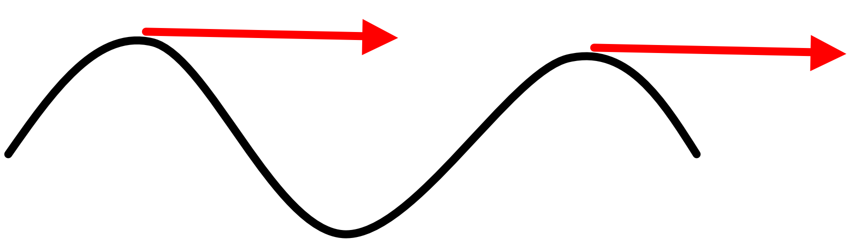
$$F_{\parallel} = eE_{\perp}\sin\theta$$



$W_{\parallel} \sim s^{-1/3}$

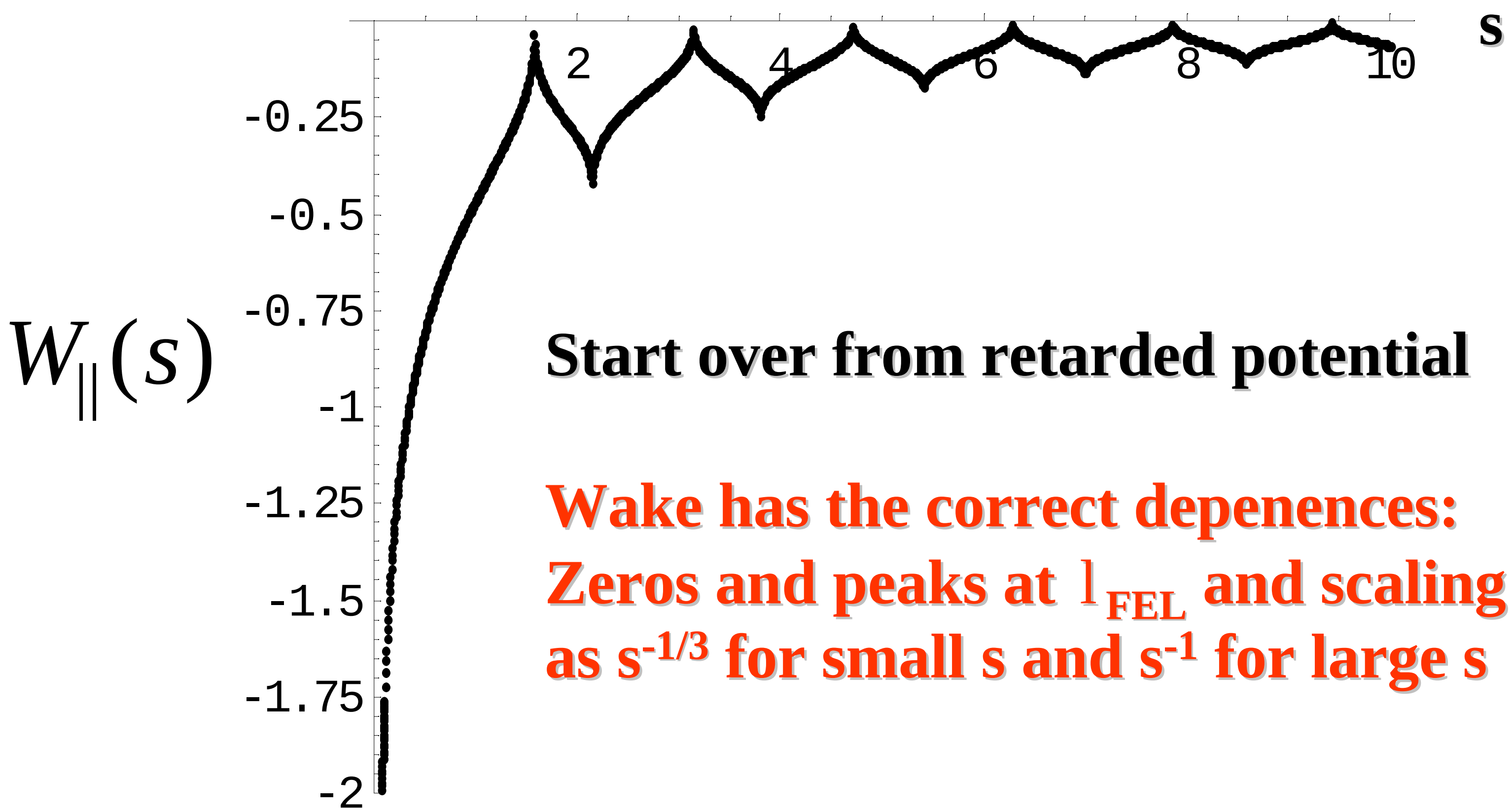
$s^{-1/3}$ comes from retarded position

- **Wiggler:**



- $W_{\parallel} = 0$, for $s = n\,\lambda_{FEL}$;
- $W_{\parallel} = \max$, for $s = \lambda_{FEL}/2$;
- $W_{\parallel} \sim s^{-1/3}$, for $s \rightarrow 0$;
- $W_{\parallel} \sim s^{-1}$, for $s >> \lambda_{FEL}$.

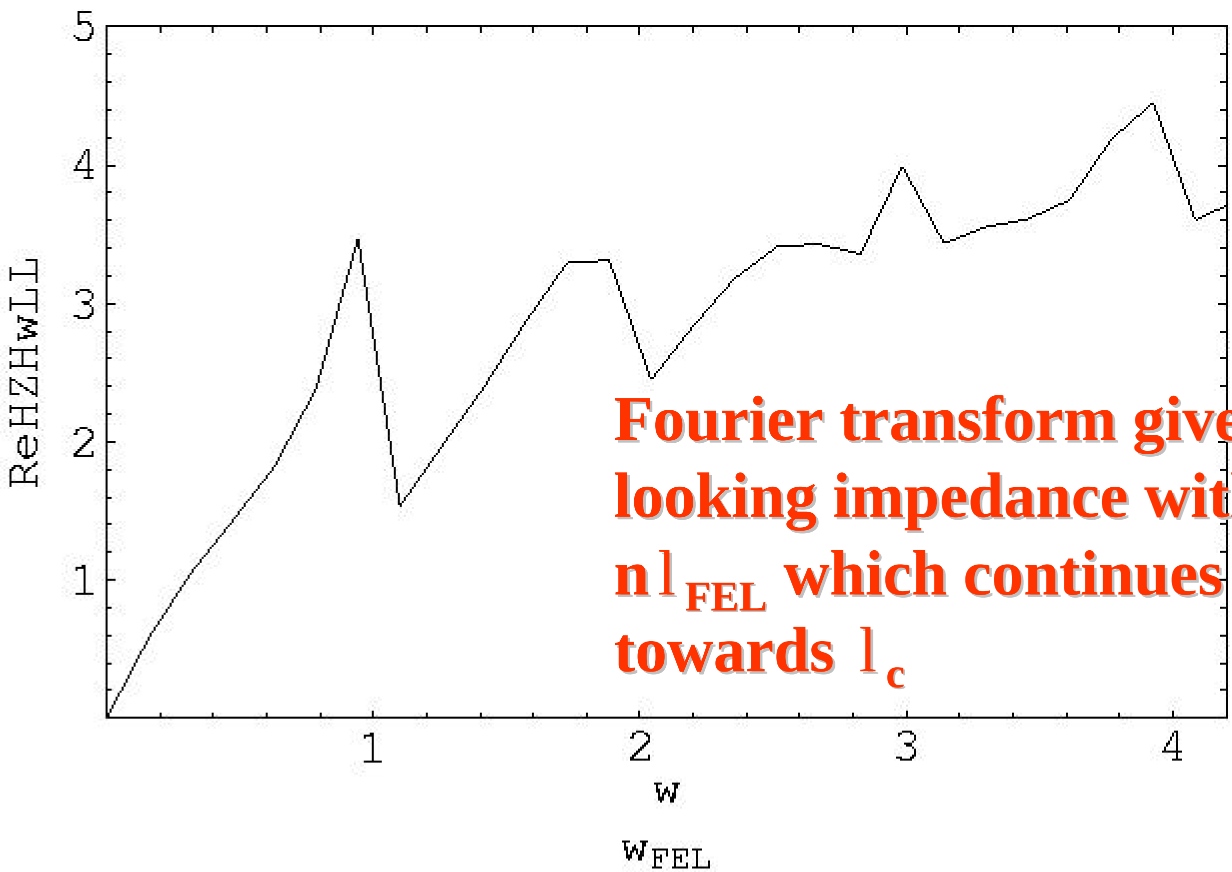
Universal Numerical Wake Potential for Large K



Start over from retarded potential

Wake has the correct depenences: Zeros and peaks at λ_{FEL} and scaling as $s^{-1/3}$ for small s and s^{-1} for large s

Impedance for Wiggler



Fourier transform gives reasonable looking impedance with peaks at $n l_{FEL}$ which continues to increase towards l_c

Connect to FEL instability

- **1-D theory predicts very short-gain length**
- **However, minimal beam-radiation overlap**

- **TDA simulation gives**

$$L_G^{wiggler} \approx 31\,\mathrm{m}$$

Single pass isn't dangerous, but how about multi-turn? Need numerical simulations with momentum compaction for ring.

Summary

- Instability threshold is uncomfortably close!
  - Factor of 2~3 without wiggler
  - Factor of 1 with incorrect wiggler
  - ??
- Calculated a wake and impedance function for the wiggler
  - Transient effects?
- Need to complete calculation for threshold
- Need to understand operation above threshold
  - No effect seen in ALS and NSLS on beam, but it also used to be stated that the microwave instability was benign!
- Need numerical simulations

Forthcoming topics

- **Based on the numerical full-range wake potential, we will try to get the full-range impedance numerically and compare analytical results at**
  - **Small and large frequency limits;**
  - **Compare Re[Z(w)] with the wiggler radiation power**
- **Compute growth rate as a function of frequency;**
  - **Away from singularity; (Logarithmic divergence)**
  - **Near singularity. (Does energy spread suppress it?)**
- **Try to understand the following topics:**
  - **Finite undulator length;**
  - **Finite electron beam size;**
  - **Gaps between the undulators, de-phasing!**
- **Try to understand the transition from CSR instability to FEL instability;**
- **Complete the code for simulation!**