Quantum Mechanical Limits on Beam Demagnification & Luminosity

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- minimum spot size
- phase-space density
- synchrotron radiation: Oide limit
- beamstrahlung
- ultimate luminosity

Thanks to Ralph Assmann, John Jowett, Francesco Ruggiero!

parameter	symbol	TESLA	NLC	CLIC
bunch population [10^{10}]	N_b	2	0.8	0.4
DR energy [GeV]	E_{DR}	5	1.98	2.42
emittance from DR [nm]	$\gamma\epsilon_{x,y,\mathrm{DR}}$	8000, 20	3000, 30	620, 5
DR bunch length [mm]	σ_z	6	3.6	1.2
DR energy spread [10^{-3}]	σ_E/E	1	0.9	1.1
DR beta [m]	$\beta_{y,\mathrm{DR}}$	60	4	4
FF emittance [nm]	$\gamma\epsilon_{x,y,\mathrm{FF}}$	10000, 30	3600, 40	680, 10
IP beam energy [GeV]	E^*	250	250	1500
IP beta [mm]	$\beta^*_{x,y}$	15, 0.4	8, 0.11	6, 0.07
IP spot size w/o pinch [nm]	$\sigma^*_{x,y}$	554, 5.0	243, 3.0	67, 0.7
free length from IP [m]	l^*	3.0	3.5	4.3
Upsilon	Υ	0.05	0.13	8.3
BS photons / e^-	N_γ	1.56	1.26	2.32

Diffraction Limited Spot Size

(inspired by K.-J. Kim, QABP 2000, Capri)

classical particle beam

$$\sigma_y = \sqrt{\epsilon_y \left(\beta_y^* + \frac{s^2}{\beta_y^*}\right)}$$

light (Z_R: Rayleigh length)

$$\sigma_y^\gamma = \sqrt{\frac{\lambda}{4\pi} \left(Z_R + \frac{s^2}{Z_R}\right)}$$

quantum particle

$$\sigma_y^{QM} = \sqrt{\frac{\lambdabar_e}{2\gamma} \left(\beta_y^* + \frac{s^2}{\beta_y^*}\right)}$$

QM important if $\gamma\epsilon_y \to \lambdabar_e/2 \approx 0.2$ pm; minimum spot size 5–30 pm for CLIC, NLC, TESLA.

Shall We Describe Beam by Wigner Function?

$$W(x,p) = \frac{1}{\hbar} \int_{-\infty}^{\infty} \exp\left(\frac{i}{\hbar} p y\right) \, \phi\left(x - \frac{y}{2}\right) \phi^* \left(x + \frac{y}{2}\right) \, dy$$

where $\phi(x)$ wave function; e.g., for uncorrelated Gaussian wave packet:

$$W(x,p,x_0,p_0) = \frac{1}{2\pi\sigma_x\sigma_p} \exp\left(-\frac{1}{2}\left[\frac{(x-x_0)^2}{\sigma_x^2} + \frac{(p-p_0)^2}{\sigma_p^2}\right]\right)$$

where $\sigma_x \sigma_p = \hbar/2$; W corresponds to particle density, but, for general ϕ, W is not a positive function!

temporal evolution of $W(x,p,t)$: Wigner-Moyal equation $\equiv$ Liouville equation (for quadratic potentials)

Phase Space Density

each phase space cell of dimension h^3 can accommodate 1 polarized electron

$\rightarrow$ density limit:

$$\rho_{ps} \equiv \frac{N}{\gamma^3 \epsilon_x \epsilon_y \epsilon_z} \leq \frac{1}{\lambda\!\!\!^{-}{}_e^3}$$

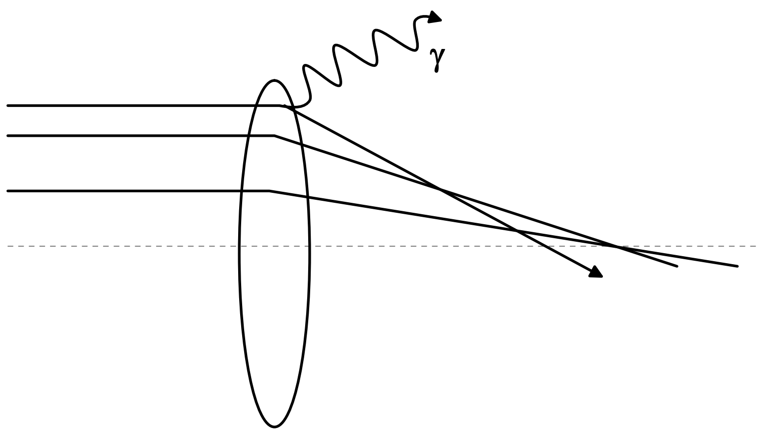
parameter	symbol	TESLA	NLC	CLIC
DR design density [m^{-3}]	ρ_{ps}	2×10^{23}	7×10^{23}	2×10^{25}
density limit [m^{-3}]	$1/\lambda\!\!\!^{-}{}_e^3$	7×10^{34}	7×10^{34}	7×10^{34}

we are far away from the limit thanks to longitudinal emittance!

Synchrotron Radiation in Final Quadrupole(s)

(K. Oide, PRL 61, 1713 (1988))

particles emit synchrotron radiation photons, lose energy, acquire different focal lengths → spot-size blow up



energy loss per unit length

$$\frac{d\delta}{ds} \approx \frac{2}{3} r_e \gamma^3 \frac{K^2 {l^*}^2 \epsilon_y}{\beta_y^*}$$

total number of photons emitted

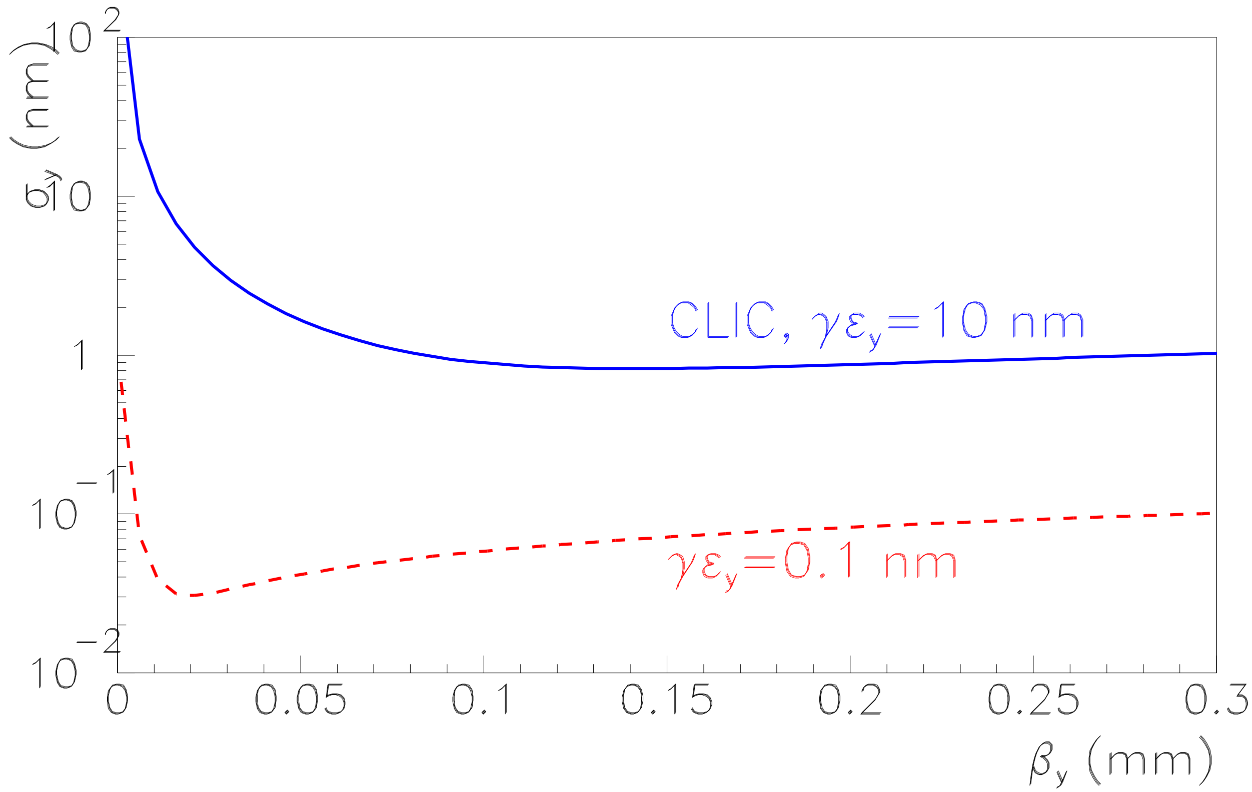
$$N \approx \frac{5}{2\sqrt{3}} \alpha \gamma \theta \approx \frac{5}{2\sqrt{3}} \alpha \gamma K l^* \sqrt{\epsilon_y / \beta_y^*}$$

photon energy squared per unit length

$$\frac{d\delta}{ds} \approx \frac{55}{24} r_e \lambda\!\!\!^{-}_e \gamma^5 \frac{K^3 {l^*}^3 \epsilon_y^{3/2}}{{\beta_y^*}^{3/2}}$$

Synchrotron Radiation in Final Quadr. Cont'd

$$\sigma_y^{*2} = \beta_y^* \epsilon + \frac{110}{3\sqrt{6\pi}} r_e \lambda\!\!\!^{-}_e \gamma^5 F(K_Q, L_q, l^*) \left(\frac{\epsilon_y^*}{\beta_y^*}\right)^{5/2}$$



Synchrotron Radiation in Final Quadrupole(s) Cont'd

optimum choice of β_y^*,

$$\beta_y^* = \left(\frac{275}{3\sqrt{6\pi}} r_e \lambda\!\!\!^{-}_e F(K_Q, L_q, l^*) \right)^{2/7} \gamma (\gamma \epsilon_y)^{3/7} ,$$

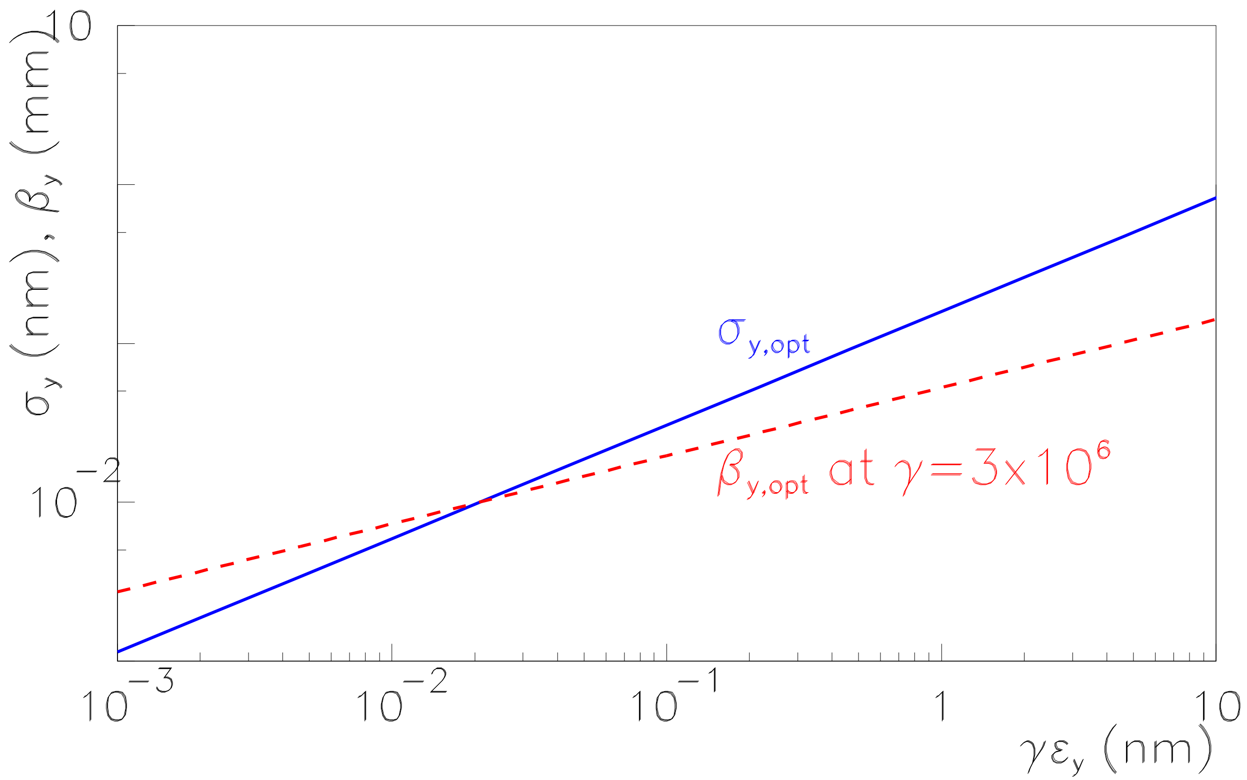
increases linearly with γ!

Minimum spot size

$$\sigma^*_{y_{\min}} = \left(\frac{7}{5} \right)^{1/2} \left(\frac{275}{3\sqrt{6\pi}} r_e \lambda\!\!\!^{-}_e F(K_Q, L_q, l^*) \right)^{1/7} (\gamma \epsilon_y)^{5/7}$$

independent of γ!

Synchrotron Radiation in Final Quadr. Cont'd



Minimum emittance $\gamma\epsilon_y \approx \lambda\!\!\!^{-}_e/2 \rightarrow$ minimum

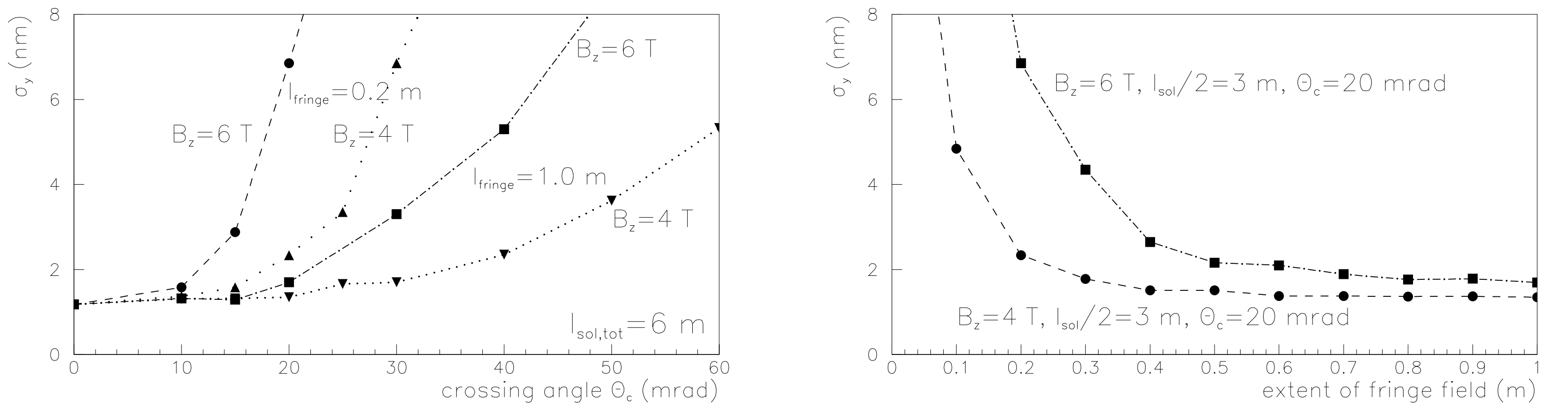
Oide spot size $\sigma^*_{y_{\rm min}} \approx 1.3$ pm! $(F \approx 5.4)$

Similar Effect: Synchrotron Radiation in Solenoid Field with Crossing Angle

Effect of SR in solenoid body computed by J. Irwin:

$$\frac{\Delta\sigma_y^{*\,2}}{\sigma_y^*} = \frac{c_u r_e \bar{\lambda}_e \gamma^5}{\sigma_y^{*\,2}} \int ds R_{36}(s)^2 \left|\frac{1}{\rho(s)}\right|^3 = \frac{1}{20}\frac{c_u r_e \bar{\lambda}_e}{\sigma_y^{*\,2}}\left(\frac{B_s\theta_c l^*\gamma}{2B\rho}\right)^5 .$$

A larger effect can arise from the fringe field of the solenoid.



Simulated vertical spot size σ_y^* at the CLIC collision point vs. θ_c (left) and vs. length of fringe field (right), considering solenoid fields of 4 and 6 T. (D. Schulte & F.Z., PAC 2001)

Oide Limit - Possible Remedies

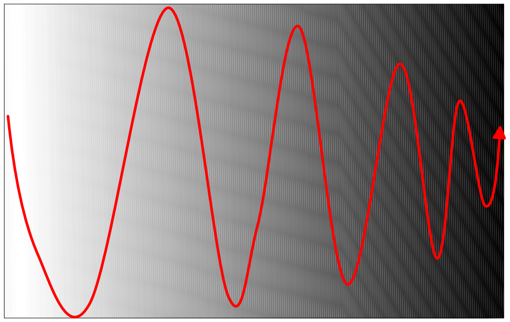
- photon statistics $\rightarrow$ rms spot size overestimates the luminosity loss (Hirata, Oide, Zotter, Phys. Lett. B 224, 437 (1989))
- adjust strength of quadrupole to compensate for average energy loss
- install octupoles near final quadr. to compensate average energy loss at various amplitudes (P. Raimondi)
- suppress the radiation with $\rho/\gamma \gg \beta$ (?)
- suppress the radiation by using 'ultra-dense' beam (?)
- strong adiabatic focusing

Adiabatic Focuser

(P. Chen, K. Oide, A.M. Sessler, S.S. Yu, PRL 64, 1231 (1999))

adiabatic focusing

$$K(s) = \frac{1+\alpha_0^2}{\beta(s)^2}$$



focusing strength increases with s as $1/\beta^2$;

amplitude of lower-energy particle never exceeds that of reference particle; holds true for particles which emit radiation;

energy loss in 3 regimes:

$$\frac{d\gamma}{ds} = -\frac{2}{3}\frac{\alpha}{\lambda\!\!\!^{-}_e} \times \begin{cases} \Upsilon^2, & \Upsilon \leq 0.2 \quad \text{(classical)} \\ 0.2\Upsilon, & 0.2 \leq \Upsilon \leq 22 \quad \text{(transition)} \\ 0.556\Upsilon^{2/3}, & 22 \leq \Upsilon \quad \text{(quantum)} \end{cases}$$

where $\Upsilon = \gamma^2 \lambda\!\!\!^{-}_e/\rho$; trick is to exploit the quantum regime!

Adiabatic Focuser Cont'd

limitation: fractional energy loss must remain small!

→ maximum emittance for transition regime:

$$\gamma\epsilon_{\rm trans} \leq \frac{5^4 6 \lambda\!\!\!^{-}_e}{\alpha} \frac{\alpha_0^3}{(1+\alpha_0^2)^2}\;;$$

→ maximum emittance for quantum regime:

$$\gamma\epsilon_{\rm quant} \leq \frac{15^3}{2^3 22} \frac{\lambda\!\!\!^{-}_e}{\alpha} \frac{\alpha_0^3}{(1+\alpha_0^2)^2}\;;$$

use $\alpha_0 = \sqrt{3}$; find critical emittance to enter quantum regime:

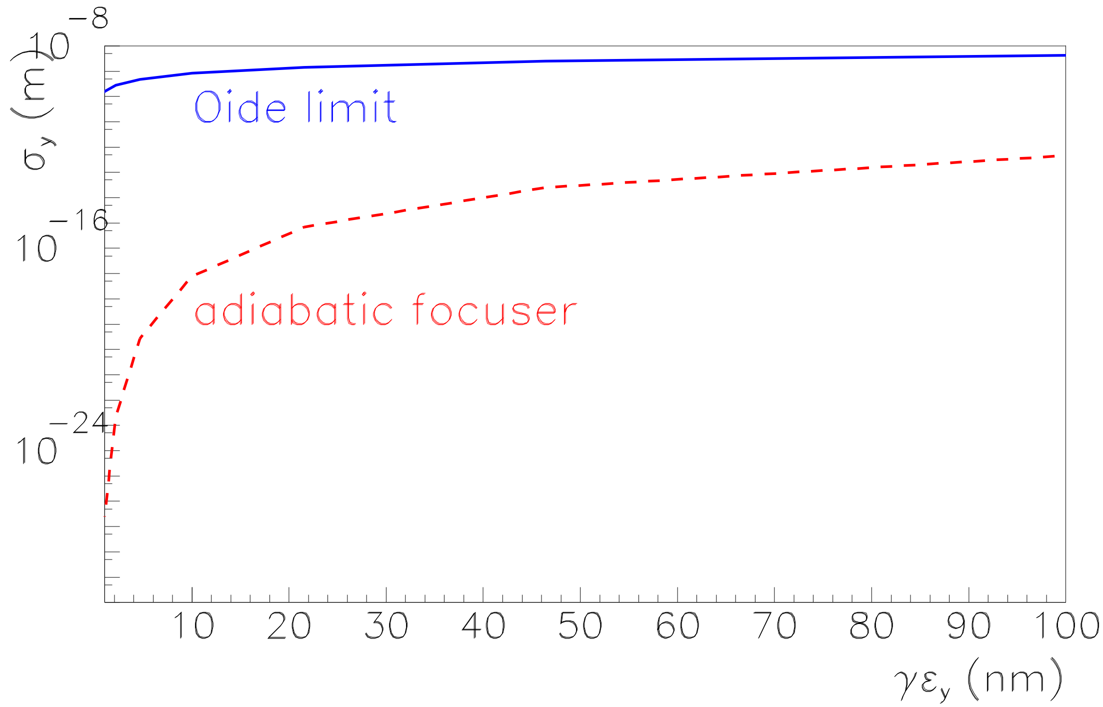
$$\gamma\epsilon_c = \frac{3^{3/2} 15^3}{2^3 4^2 22} \lambda\!\!\!^{-}_e \alpha^3 \approx 6.2\times 10^{-6}\ {\rm m}\,;$$

minimum spot size

$$\sigma^*_{y\,{\rm min,AF}} = \left(\frac{2}{11}\lambda\!\!\!^{-}_e (\gamma\epsilon_y)^2\right)^{1/3} \exp\left(-3\left(\frac{3^{3/2}}{16}\frac{\lambda\!\!\!^{-}_e}{\alpha^3\gamma\epsilon_y}\right)^{1/2}\right)$$

Adiabatic Focuser Cont'd

promise of extremely small spot sizes

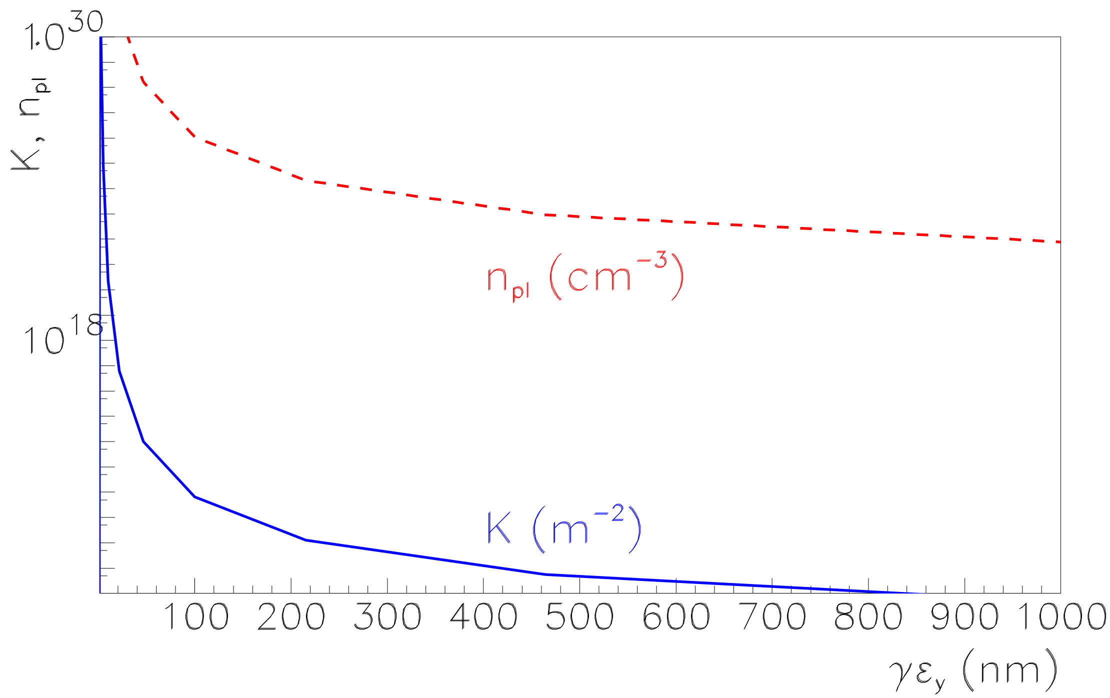


for $\gamma\epsilon_y = 10$ nm, adiabatic focuser reaches

$\sigma^*_{y_{\min,\mathrm{AF}}} = 10^{-9}$ nm! (e^- wave nature not cons.)

Adiabatic Focuser Cont'd

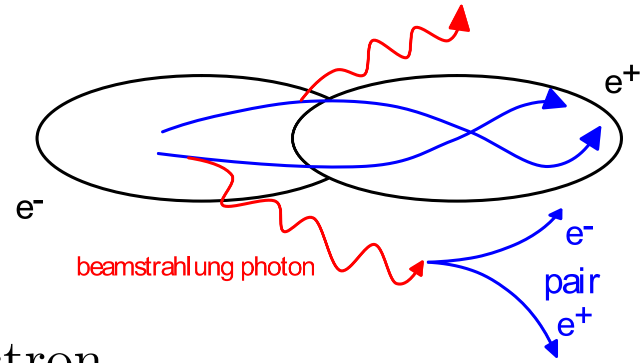
a problem with this approach



final gradient K and corresponding plasma density $n_{\rm pl} \approx \gamma K/(2\pi r_e)$; 'plasma' may become denser than a solid!

IP Physics: Beamstrahlung & L Spectrum

Upsilon parameter



$$\Upsilon_{\rm avg} = \left\langle \frac{2\hbar\omega_c}{3E} \right\rangle \approx \frac{5}{6} \frac{\gamma {r_e}^2 N}{\alpha \sigma_z (\sigma_x + \sigma_y)}$$

Number of beamstrahlung photons/electron

$$N_\gamma \approx \frac{5\alpha\sigma_z}{2\gamma \lambda\!\!\!^{-}_e} \frac{\Upsilon}{(1+\Upsilon^{2/3})^{1/2}} \approx 2 \, \frac{\alpha r_e N}{\sigma_x + \sigma_y} \, ;$$

last approximation applies if Υ is small ($\Upsilon \leq 1$).

Fraction of the luminosity at the design center-of-mass energy:

$$\Delta\mathcal{L}/\mathcal{L} \approx 1/N_\gamma^2 (1 - e^{-N_\gamma})^2$$

Average energy loss

$$\delta_B \approx \frac{1}{2} N_\gamma \Upsilon \frac{(1+\Upsilon^{2/3})^{1/2}}{(1+(1.5\Upsilon)^{2/3})^2}$$

Ultimate Luminosity

$$L = \frac{f_{\rm rep} n_b N_b^2}{4\pi \sigma_x^* \sigma_y^*} = \frac{1}{4\pi} \frac{P_{\rm wall}\eta}{E_b} \frac{N_b}{\sigma_x^* \sigma_y^*}$$

where $\eta = P_{\rm beam}/P_{\rm wall} = f_{\rm rep} N_b n_b / P_{\rm wall}$.

(1) ignore beamstrahlung & Oide effect; assume minimum emittances: $\gamma\epsilon_{x,y,z} \approx \lambda\!\!\!^{-}_e/2$, $\beta^*_{x,y} \approx \sigma_z$, and $\sigma_z \approx (\gamma\epsilon_z)/\gamma/(\Delta p/p)_{\rm rms}$:

$$L_1 = \frac{1}{4\pi} \frac{P_{\rm wall}\eta N_b}{E_b} \frac{4\gamma^2}{\lambda\!\!\!^{-2}_e} \left(\frac{\Delta p}{p}\right)_{\rm rms}$$

where, e.g., $(\Delta p/p)_{\rm rms}$ may be final-focus bandwidth (≈ 0.0028)

(2) more realistic: maintain constant N_γ: $N_b/\sigma_x \approx$ const., $\gamma\epsilon_{y,z} \approx \lambda\!\!\!^{-}_e/2$, $\beta^*_y \approx \sigma_z$, $\sigma_z \approx (\gamma\epsilon_z)/\gamma/(\Delta p/p)_{\rm rms}$, and small Υ:

$$L_2 \approx \frac{1}{8\pi\alpha} \frac{P_{\rm wall}\eta N_\gamma}{E_b} \frac{2\gamma}{\lambda\!\!\!^{-}_e} \sqrt{(\Delta p/p)_{\rm rms}}$$

potential luminosity increase over design for the two cases:

$$H_{L,1} = \frac{L_1}{L_{\rm design}} \approx \frac{4\gamma^2}{\lambda_e^2} \sigma_x^{*,{\rm design}} \sigma^*_{y,{\rm design}} (\Delta p/p)_{\rm rms} \left(\frac{\gamma}{\gamma_0}\right)^2$$

$$H_{L,2} = \frac{L_2}{L_{\rm design}} \approx \frac{2\gamma}{\lambda_e} \sigma^*_{y,{\rm design}} \sqrt{(\Delta p/p)_{\rm rms}} \left(\frac{\gamma}{\gamma_0}\right)$$

luminosity should increase as γ^2 from present design $\rightarrow$ energy reach

parameter	symbol	TESLA	NLC	CLIC
case (1) for $\gamma = \gamma_0$	$H_{L,1}$	1.3×10^{18}	3.3×10^{17}	7.7×10^{17}
energy reach		∞	∞	∞
case (2) for $\gamma = \gamma_0$	$H_{L,2}$	1.1×10^{8}	6.4×10^{7}	(8.9×10^{7})
energy reach [GeV]		2.7×10^{19}	1.6×10^{19}	(1.3×10^{20})

we can reach the Planck scale (1.2×10^{19} GeV)!

(J. Irwin, private communication, 1996).

Conclusions

- quantum nature of electrons allows focusing the spot sizes down to 1 pm or less (Picobeam workshop?)
- from fundamental principles, even the Planck scale can be reached with $L \propto \gamma^2$!
- but synchrotron radiation in final quad.'s (Oide effect) & beamstrahlung at IP constrain already present designs
- new ideas and approaches are welcome!