Coherent Synchrotron Radiation effect in damping rings*

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Abstract

Coherent Synchrotron Radiation (CSR) can play an important role by not only increasing the energy spread and emittance of a beam, but also leading to a potential instability. Previous studies of the CSR induced longitudinal instability were carried out for the CSR impedance due to dipole magnets. In this paper, the instability due to the CSR impedance from a wiggler is studied assuming a large wiggler parameter K. The primary consideration is a low frequency microwave-like instability in the damping rings of several linear collider projects. The threshold is determined by the instability with the longest possible wavelength.

1 INTRODUCTION

Many modern advanced accelerator projects [1]-[3] call for short bunches with low emittance and high peak current where coherent synchrotron radiation (CSR) effects may play an important role. CSR is emitted at wavelengths longer than or comparable to the bunch length whenever the beam is deflected [4, 5]. The stringent beam requirements needed for short wavelength SASE Free-Electron Lasers have led to intensive theoretical and experimental studies [6]-[11] over the past a few years where the focus has been on the magnetic bunch compressors required to obtain the high peak currents. In addition to these single-pass cases, it is also possible that CSR might cause a microwave-like beam instability in storage rings. A theory of such an instability in a storage ring has been recently proposed in Ref. [12] with experimental evidence published in [13]. Other experimental observations [14]-[17] may also be associated with a CSR-driven instability as supported by additional theoretical studies [18, 19].

The previous study of the CSR induced instability assumed that the impedance is generated by the synchrotron radiation of the beam in the storage ring bending magnets [12]. In some cases (e.g. the NLC damping ring [20]), a ring will include magnetic wigglers which introduce an additional contribution to the radiation impedance. The analysis of the microwave instability in such a ring requires knowledge of the impedance due to the synchrotron radiation in the wiggler. We calculate the wakefield and impedance in a wiggler with a large parameter K [21] using results from some earlier studies [22, 23]. We then study the impact of the wiggler synchrotron radiation impedance on the beam longitudinal dynamics, both in the wiggler itself, and in rings with dipoles and wigglers [24].

2 THEORY

We focus on the longitudinal dynamics and study a thin coasting beam. In a wiggler, the beam can be described by a longitudinal distribution function $\rho(\nu, s, z)$. The positive direction for the internal coordinate s is pointing to the forward. The relative energy offset of a particle with energy E with respect to the nominal energy E_0 is expressed as $\nu = (E - E_0)/E_0$. The position of the reference particle in the beam line is $z = c\,t$ with c to be the speed of light in vacuum. Following Ref. [12], we use a 1-D Vlasov equation to describe the evolution of the longitudinal distribution function $\rho(\nu, s, z)$,

$$\begin{aligned} 0 &= \frac{\partial \rho}{\partial z} - \eta\,\nu\,\frac{\partial \rho}{\partial s} \\ &- \frac{r_0}{\gamma}\frac{\partial \rho}{\partial \nu}\int_{-\infty}^{\infty} d\,\nu' \int_{-\infty}^{s} d\,s'\,w(s-s')\,\rho(\nu', s', z)\ , \end{aligned} \qquad (1)$$

where, the slippage factor is defined as $\eta \equiv \partial^2 s/\partial\nu\,\partial z = \alpha - 1/\gamma^2$, with α to be the momentum compaction factor. The wake Green function $w(s)$ describes the interaction of two particles due to the synchrotron radiation, and $w(s) \neq 0$ for $s > 0$; $w(s) = 0$ for $s < 0$. In Eq. (1), $r_0 \approx 2.82 \times 10^{-15}$ m is the classical electron radius, and γ is the Lorentz factor. In the following, we assume the equilibrium energy distribution function is a Gaussian, i.e., $\rho_0 = n_0/(\sqrt{2\pi}\,\nu_0) \times \exp(-\nu^2/2\,\nu_0^2)$, where n_0 is the linear density, i.e., the number of particles per unit length.

The instability is then determined by the following dispersion relation

$$1 = -\frac{i\,Z(k)\,\Lambda}{\sqrt{2\,\pi}\,k}\int_{-\infty}^{\infty} d\,p\,\frac{p\,e^{-\frac{p^2}{2}}}{\Omega \pm p}\ . \qquad (2)$$

Here, $\Lambda = n_0\,r_0/(|\eta|\,\gamma\,\nu_0^2)$; $\Omega = \omega/(c\,k\,|\eta|\,\nu_0)$, $p = \nu/\nu_0$, and $Z(k) = \int_0^\infty d\,s\,w(s)\,\exp\{-\,i\,k\,s\}$, is the CSR impedance. The upper (lower) sign in Eq. (2) refers to the case of a positive (negative) η.

In a damping ring, there are both dipoles and wigglers. The corresponding CSR impedance is the summation of the impedance from the dipoles and the wigglers:

$$Z(k) = Z_D(k)\frac{\Theta\,R}{C} + Z_W(k)\frac{L_W}{C}\ , \qquad (3)$$

where, R, Θ, L_W and C are the dipole bending radius, the total bending angle, the wiggler total length and the damping ring circumference.

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In our model, we use the steady state impedance. For a dipole, the steady state CSR impedance is [7]

$$Z_D(k) = -\,i\,A\,\frac{k^{1/3}}{R^{2/3}}\;, \tag{4}$$

with $A = 3^{-1/3}\,\Gamma(2/3)(\sqrt{3}\,i - 1)$.

The wiggler impedance $Z_W(k)$ is computed in Ref. [21]. For the reasons discussed below, the instability threshold is lowest at relatively low frequency. The asymptotic behavior as $k \to 0$ of Z_W is [21]

$$Z_W(k) = \pi\,k_w \frac{k}{k_0}\left[1 - \frac{2\,i}{\pi}\,\log\left(\frac{k}{k_0}\right)\right]\;. \tag{5}$$

This is accurate enough for the low-frequency region $k \in [0, 0.1\,k_0]$, where $k_0 \equiv 4\gamma^2 k_w/K^2$ is the wiggler fundamental radiation wavenumber.

3 RESULTS

We will numerically solve the dispersion relation in Eq. (2), and study the damping rings of the proposed linear collider projects [1, 2], where damping wigglers are used. The parameters are given in Table 1. However, before using Eq. (2) to calculate the threshold, let us consider a scaling analysis. According to Eq. (4), the dipole CSR impedance scales as $Z_D(k) \propto k^{1/3}$, while, according to Eq. (5), the wiggler CSR impedance scales as Re($Z_W(k)$)$\propto k$, and Im($Z_W(k)$)$\propto k\log(k)$, which is a weaker scaling than k. Hence, the CSR induced energy modulation has a scaling no-stronger than k. On the other hand, the slippage effect is linear proportional to k according to the second term in Eq. (1), or more clearly in the denominator of the dispersion relation in Eq. (2). In the beam, there is a finite energy spread, which will produce a phase mixing due to the slippage effect. Such phase mixing effect will destroy the density modulation due to the CSR induced energy modulation. This phase mixing effect is more serious for short wavelength perturbations, and the resulting damping is proportional to k. As we explained above, the growth due to the CSR impedance is weaker than the linear scaling of the phase mixing effect. Hence, it is expected that the threshold is determined by the perturbation with the longest possible wavelength.

In a real machine, the vacuum chamber causes an exponential suppression of the radiation at wavelengths λ greater than a "shielding cutoff" [4]

$$\lambda_c \le 4\sqrt{2}r(r/R)^{1/2}\;, \tag{6}$$

where, R is the dipole bending radius, and r is the vacuum chamber half height. The numerical coefficient of Eq. (6) assumes that the vacuum chamber is made up of two infinitely wide plates. Different cross sections give different numerical factors [27]. Given the previous discussion, the threshold will be the lowest at the "shielding cutoff" wavelength.

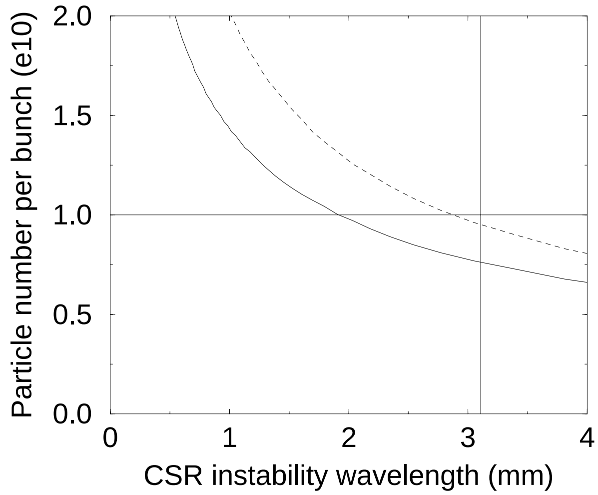

Figure 1: The threshold as a function of the CSR wavelength for the KEK ATF damping ring [26]. Plotted on the vertical axis is the number of particles per bunch, in units of 10^{10}. Plotted on the horizontal axis is the CSR instability wavelength in units of mm. The dashed line is the result for the dipoles only, while the solid line takes into account of the contributions from the dipoles and the wigglers. The vertical straight line is the approximate critical wavelength beyond which the radiation is exponentially suppressed. The horizontal straight line is the nominal particle number per bunch, i.e., 1×10^{10}.

For the KEK ATF prototype damping ring [26], with the parameters in Table 1, the cut-off wavelength would be about $\lambda_c \approx 3.1$ mm according to Eq. (6). In Fig. 1, we plot the threshold as a function of the perturbation wavelength. It is clearly seen that the threshold current decreases as we approach the longer wavelength perturbations; a result expected according to our scaling analysis. Taking the dipole CSR impedance alone, for the single bunch charge, the instability sets in for perturbations with wavelengths $\lambda > 2.8$ mm. Adding the wiggler CSR impedance, the electron beam would be unstable for perturbations with wavelengths $\lambda > 1.9$ mm.

For the NLC main damping ring [20], with the nominal current, we find that perturbations with wavelengths $\lambda > 3.5$ mm are not stable due to the dipole CSR impedance alone. Adding the CSR impedance from the wiggler causes perturbations with wavelengths $\lambda > 2.6$ mm to be unstable. In Fig. 2, we plot the threshold as a function of the perturbation wavelength. For the TESLA damping ring [25], the impedance from the dipoles and wigglers will not drive the instability for perturbations with wavelengths $\lambda < 5$ mm. Based on the parameters in Table 1 and according to Eq. (6), we computed the "shielding cutoff" wavelengths. At these cutoff wavelengths, the threshold currents are summarized in Table 1.

	NLC	TESLA	ATF
Circumference C/km	0.3	17	0.14
Dipole radius R/m	5.5	80	5.73
Total bending angle $\Theta/2\pi$	1	5/3	1
Momentum compaction $\alpha/10^{-4}$	2.95	1.2	19
Energy E/Gev	1.98	5	1.3
Energy rms spread $\nu_0/10^{-4}$	9.09	9	6
Bunch rms length σ_z/mm	3.6	6	5
Particles in a bunch $N_e/10^9$	7.5	20	10
Wiggler peak field B_w/T	2.15	1.5	1.88
Wiggler period λ_w/m	0.27	0.4	0.4
Wiggler total length L_w/m	46.24	432	21.2
Cross section half height r/cm	1.6	2	1.2
Cutoff wavelength λ_c/mm	4.9	1.8	3.1
Threshold at cutoff (wiggler off) $N_t/10^9$	6.0	274.4	9.5
Threshold at cutoff (wiggler on) $N_t/10^9$	5.2	245.6	7.6

Table 1: Parameters and results for the NLC main damping ring [20], the TESLA damping ring [25], and the KEK ATF prototype damping ring [26].

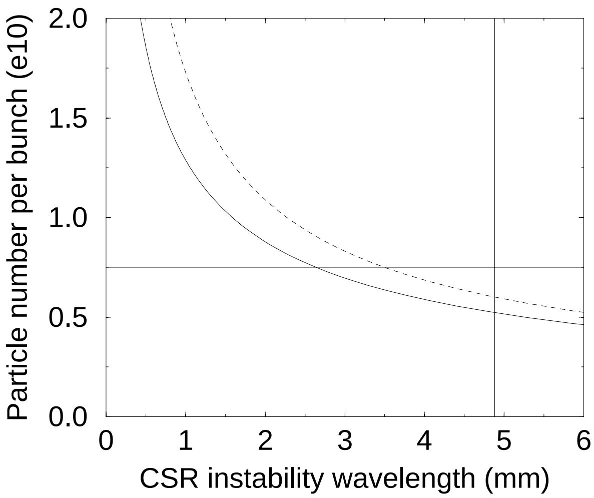

Figure 2: The threshold as a function of the CSR wavelength for the NLC main damping ring [20]. Plotted on the vertical axis is the number of particles per bunch, in units of 10^{10}. Plotted on the horizontal axis is the CSR instability wavelength in units of mm. The dashed line is the result for the dipoles only, while the solid line takes into account of the contributions from the dipoles and the wigglers. The vertical straight line is the approximate critical wavelength beyond which the radiation is exponentially suppressed. The horizontal straight line is the nominal particle number per bunch, i.e., 0.75×10^{10}.

4 DISCUSSION

The theory in this paper is for an ideal ring with distributed CSR impedance from dipoles and wigglers. The CSR impedance used in this paper is the steady state result and a costing beam assumption is adopted in the theory. In reality, there are other sources of impedance, which will initiate some energy modulation and density modulation in the beam. The cutoff calculation is also based on an ideal ring with vacuum chamber consisting of infinitely wide plates. Further theoretical work should include folding the full radiation spectrum with the cutoff into the impedance. There is also a question about whether a mode can copropagate with the electron beam. All these uncertainties in the theory need further exploration. An experiment in KEK ATF prototype damping ring will help verify the theory and clarify the cutoff issues.

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